Advanced Functions, Grade 12, University Preparation (MHF 4U)

2023-2024 Course Outline

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Developed from *The Ontario Curriculum Grade 11 and 12 Mathematics (revised)*, published 2007

Credit Value: 1.0

Prerequisite Courses: Functions, Grade 11, University Preparation, or Mathematics for College Technology, Grade 12, College Preparation

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COURSE DESCRIPTION

This course extends students' experience with functions. Students will investigate the properties of polynomial, rational, logarithmic, and trigonometric functions; develop techniques for combining functions; broaden their understanding of rates of change; and develop facility in applying these concepts and skills. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended both for students taking the Calculus and Vectors course as a prerequisite for a university program and for those wishing to consolidate their understanding of mathematics before proceeding to any one of a variety of university programs.

Credit Value: 1.0

Prerequisite courses: Functions, Grade 11, University Preparation, or Mathematics for College Technology, Grade 12, College Preparation

Mathematical process expectations

The mathematical processes are to be integrated into student learning in all areas of this course.

Throughout this course, students will:

- develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
- develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
- demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
- select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
- make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
- create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;
- communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

CURRICULUM EXPECTATIONS

Exponential and Logarithmic Functions

By the end of this course, students will:

- demonstrate an understanding of the relationship between exponential expressions and logarithmic expressions, evaluate logarithms, and apply the laws of logarithms to simplify numeric expressions;
- identify and describe some key features of the graphs of logarithmic functions, make connections among the numeric, graphical, and algebraic representations of logarithmic functions, and solve related problems graphically;
- solve exponential and simple logarithmic equations in one variable algebraically, including those in problems arising from real-world applications.

Trigonometric Functions

By the end of this course, students will:

- demonstrate an understanding of the meaning and application of radian measure;

- make connections between trigonometric ratios and the graphical and algebraic representations of the corresponding trigonometric functions and between trigonometric functions and their reciprocals, and use these connections to solve problems;
- solve problems involving trigonometric equations and prove trigonometric identities.

Polynomial & Rational Functions

By the end of this course, students will:

- identify and describe some key features of polynomial functions, and make connections between the numeric, graphical, and algebraic representations of polynomial functions;
- identify and describe some key features of the graphs of rational functions, and represent rational functions graphically;
- solve problems involving polynomial and simple rational equations graphically and algebraically;
- demonstrate an understanding of solving polynomial and simple rational inequalities.

Characteristics of Functions

By the end of this course, students will:

- demonstrate an understanding of average and instantaneous rate of change, and determine, numerically and graphically, and interpret the average rate of change of a function over a given interval and the instantaneous rate of change of a function at a given point;
- determine functions that result from the addition, subtraction, multiplication, and division of two functions and from the composition of two functions, describe some properties of the resulting functions, and solve related problems;
- compare the characteristics of functions, and solve problems by modelling and reasoning with functions, including problems with solutions that are not accessible by standard algebraic techniques.

COURSE OUTLINE

Unit Number	Unit Name	Topics Covered	Instructional Hours	Overall Curriculum Expectations
1	Basic Skills Review	This unit reviews the foundational concepts that have been covered in prerequisite math courses. Students revisit the definition of a function, function notation, and the key properties of functions. Students also review transformations of functions and inverse functions. The unit assessment evaluates students' ability to carry out proper communication, formatting, and technical skills in their work, all of which will be important aspects of their assignments in the remainder of the course.	6	D1, D2, D3
2	Polynomial Functions	In this unit students learn to identify and describe some key features of polynomial functions and to make connections between the numeric, graphical, and algebraic representations of polynomial functions. These concepts allow students to manipulate functions in a number of ways and apply their skills to solve real-world problems. Strategies will be employed to aid in the connection to an understanding of rates of change.	20	C1, C2, C3, C4
3	Rational Functions & Inequalities	Students begin this unit by identifying and describing some of the key features of rational functions. Students then learn to represent and manipulate these functions to solve real-life problems, graphically and algebraically. This unit also introduces the idea of inequalities and how they produce different solutions than equations.	15	C1, C2, C3, C4
4	Exponential & Logarithmic Functions	This unit begins with a review of exponential functions, their properties, and applications. This leads into discussions about a related function, the logarithmic function. From here students learn about logarithmic properties and then apply their knowledge of exponential and logarithmic functions to solve real-world problems.	18	A1, A2, A3
5	Trigonometry	This unit examines the meaning and application of radian measure. This allows students to solve more complex situations in exact values. Students will make connections between trigonometric ratios and the graphical and algebraic representations of the corresponding trigonometric functions and use these connections to solve problems involving trigonometric equations and to prove trigonometric identities.	18	B1, B2, B3
6	Trigonometric Functions & Graphs	This unit develops students understanding of trigonometry by expanding on the functions behind the trigonometric ratios. Students look at trigonometric functions and their reciprocals, examine their key properties and behaviours, and learn how they can be transformed to model a wide range of data.	18	B1, B2, B3
7	Operations on Functions	Having studied various types of functions and transformations of functions, and understood the significance of differential rates of change in functions, this final unit focuses on the theory and practice of performing arithmetic operations on entire functions, including but not limited to the algebraic, graphical and practical implications of performing those operations.	12	D1, D2, D3

TOTAL INSTRUCTIONAL HOURS………………………………………………….. 110 Hours

TEACHING & LEARNING STRATEGIES AND STRATEGIES FOR ASSESSMENT & EVALUATION OF STUDENT PERFORMANCE

Students will follow a similar pattern of instructions in all units. To begin students will be involved in the exploration of an investigation of a concept. Then they will apply what they have learned in several real-life scenarios or applications of the concept. Students will see solutions to applications after they try to solve them for themselves. Then students will complete assignments where no solutions are provided and submit these for assessment. Finally, the unit ends with a test. A wide variety of instructional strategies are used to provide learning opportunities to accommodate a variety of learning styles, interests and ability levels. Seven mathematical processes will form the heart of the teaching and learning strategies used:

- *Communicating:* To improve student success there will be several opportunities for students to share their understanding both in oral as well as written form.
- *Problem solving:* Scaffolding of knowledge, detecting patterns, making and justifying conjectures, guiding students as they apply their chosen strategy, directing students to use multiple strategies to solve the same problem, when appropriate, recognizing, encouraging, and applauding perseverance, discussing the relative merits of different strategies for specific types of problems.
- *Reasoning and proving:* Asking questions that get students to hypothesize, providing students with one or more numerical examples that parallel these with the generalization and describing their thinking in more detail.
- *Reflecting:* Modeling the reflective process, asking students how they know.
- *Selecting Tools and Computational Strategies:* Modeling the use of tools and having students use technology to help solve problems.
- *Connecting:* Activating prior knowledge when introducing a new concept in order to make a smooth connection between previous learning and new concepts and introducing skills in context to make connections between particular manipulations and problems that require them.
- *Representing:* Modeling various ways to demonstrate understanding, posing questions that require students to use different representations as they are working at each level of conceptual development – concrete, visual or symbolic, allowing individual students the time they need to solidify their understanding at each conceptual stage.

ASSESSMENT POLICY

In keeping with the Ministry of Education's document, Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools, 2010, this course will be presented to students with consideration of the overall and specific expectations established for the credit, the achievement chart in the appropriate curriculum policy document, and the guidelines for Assessment and Evaluation. The course contains both content standards (the knowledge and skills a student is expected to demonstrate throughout the course) and performance standards (the quality of student learning as reflected by the student's work toward achieving these skills).

To support student learning and to ensure that the assessment and evaluation encourage and promote student achievement as much as possible, course evaluations will be designed with a mind to being:

- balanced and equitable, with clear instructions and criteria;
- reflective of the overall and specific expectations for the course;
- ongoing and varied, allowing students to demonstrate achievement throughout the year;
- committed to include ongoing descriptive feedback giving students indications of goals and strategies for improvement; and
- supportive of student skills in assessing their own learning (for self-improvement) so that they can set personal goals and strategies (metacognition).

ASSESSMENT TYPES

This course will contain all three types of assessment recommended by the Ministry of Education.

Assessment *for* learning
The teacher will gather information about student's skill and understanding in order to plan teaching activities to maximize student achievement. In addition, the teacher will give feedback on work which is designed to help the student direct his/her efforts to particular skills or content so that he/she can improve his/her results. These assessments are generally not completed for marks, but rather for feedback, and include such things as checklists, student reflections, practice activities, and sample questions.

Assessment *as* learning
The student will be asked to demonstrate progress in developing skills and understanding of content in a way which allows him/her to set goals, reflect on work, and determine strategies for progress. These assessments may or may not be evaluated for marks and may include such things as small tests, quizzes, brief presentations, student reflections and self and peer-assessed activities.

Assessment *of* learning
The student will be asked to demonstrate that he/she has acquired the skills taught and has developed a strong understanding of the content and performance standards related to the topic. These assessments are done in preparation for moving forward to new content and performance standards or in completion of the course itself. These are assessed for marks and are used to record and report what has been learned. They include such things as unit tests, presentations, assignments, projects, and exams.

All assessments are designed to fit into one or more of the Grade 9-12 Mathematics Achievement Chart categories: Knowledge and Understanding, Thinking, Communication, and Application (see Appendix 1).

Examples of Teaching Methods and Assessment
- Teacher led structured note taking
- Student use of software to investigate topics
- student-led discovery-based activities
- Group discussions
- Online video tutorials
- Teacher observation during class time
- Practice problems
- Take home assignments
- In class formal tests/quizzes/pre-tests/check points
- Student created problems & solutions

Learning Skills & Work Habits
The development of learning skills and work habits is needed for success in school and in life. In addition to their assessment based on the achievement chart, student success also reflects a variety of specific learning skills, through which students complete course work and assessments. These learning skills are not assigned grades based on the achievement chart, or a numeric grade, but are rather indicated on the student report card using levels (excellent, good, satisfactory, needs improvement). This indicates to the student which learning skills should receive increased effort by the student in order to improve his/her learning, and which skills are helping the student achieve their academic success. The learning skills are behaviours considered essential and integral to student learning and to the evaluation of a student's achievement as he/she progresses through each course and grade. The six learning skills are listed below; for a full description, see Appendix 2.
- Responsibility
- Organization
- Independent Work
- Collaboration
- Initiative
- Self-Regulation

ASSESSMENT STRUCTURE

Student achievement is communicated formally to students and parents by means of the Provincial Report Card. The report card provides a record of the student's achievement of the curriculum expectations in the form of a percentage grade. The percentage grade represents the quality of the student's overall achievement of the expectations for the course and reflects the corresponding level of achievement as described in the achievement chart. A final grade is recorded, and a credit is granted and recorded if the student's grade is 50% or higher.

The final grade in the course is determined as follows:

Term Work *	70%
Final Examination	30%
Final Grade	100%

*Term work is based on evaluations conducted throughout the course. This portion of the grade will reflect the student's most consistent level of achievement throughout the course, although special consideration may be given to more recent evidence of achievement.

THE MATHEMATICAL PROCESSES

This course will develop the set of seven expectations that describe the mathematical processes students need to learn and apply as they work to achieve the expectations outlined within the various strands. Students should be actively engaged in applying these processes throughout the course, rather than in connection with particular strands.

The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The mathematical processes are interconnected. Problem solving and communicating have strong links to all the other processes. A problem-solving approach encourages students to reason their way to a solution or a new understanding. As students engage in reasoning, teachers further encourage them to make conjectures and justify solutions, orally and in writing. The communication and reflection that occur during and after the process of problem-solving help students not only to articulate and refine their thinking but also to see the problem they are solving from different perspectives. This opens the door to recognizing the range of strategies that can be used to arrive at a solution. By seeing how others solve a problem, students can begin to think about their own thinking (metacognition) and the thinking of others, and to consciously adjust their own strategies in order to make their solutions as efficient and accurate as possible.

The mathematical processes cannot be separated from the knowledge and skills that students acquire throughout the course. Students must problem solve, communicate, reason, reflect, and so on, as they develop the knowledge, the understanding of concepts, and the skills required in the course.

ACHIEVEMENT CHART CATEGORIES

There are four categories into which student evaluations are divided: Knowledge and Understanding, Thinking, Communication, and Application. This means that a student's evaluated work will contain marks in all, or some, of these categories as indicated by the teacher and based on the teacher's professional judgment. Students are evaluated according to the criteria established for the course, not according to the achievement of other students. Achievement of level 3 in these categories represents the provincial standard.

There are four levels of student achievement, Levels 1-4 (as well as the possibility that a student's work can be evaluated as below level 1).

See full achievement chart for Mathematics Grade 9-12 in Appendix 1.

PROGRAM PLANNING CONSIDERATIONS

PLANNING MATHEMATICS PROGRAMS FOR STUDENTS WITH SPECIAL EDUCATION NEEDS

Classroom teachers are the key educators of students who have special education needs. They have a responsibility to help all students learn, and they work collaboratively with special education teachers, where appropriate, to achieve this goal.

Special Education Transformation: The Report of the Co-Chairs with the Recommendations of the Working Table on Special Education, 2006 endorses a set of beliefs that should guide program planning for students with special education needs in all disciplines. Those beliefs are as follows:

- All students can succeed.
- Universal design and differentiated instruction are effective and interconnected means of meeting the learning or productivity needs of any group of students.
- Successful instructional practices are founded on evidence-based research, tempered by experience.
- Classroom teachers are key educators for a student's literacy and numeracy development.
- Each student has his or her own unique patterns of learning.
- Classroom teachers need the support of the larger community to create a learning environment that supports students with special education needs.
- Fairness is not sameness.

In any given classroom, students may demonstrate a wide range of learning styles and needs. Teachers plan programs that recognize this diversity and give students performance tasks that respect their particular abilities so that all students can derive the greatest possible benefit from the teaching and learning process. The use of flexible groupings for instruction and the provision of ongoing assessment are important elements of programs that accommodate a diversity of learning needs. In planning mathematics courses for students with special education needs, teachers should begin by examining the current achievement level of the individual student, the strengths and learning needs of the student, and the knowledge and skills that all students are expected to demonstrate at the end of the course in order to determine which of the following options is appropriate for the student:

- no accommodations or modifications; or
- accommodations only; or
- modified expectations, with the possibility of accommodations; or
- alternative expectations, which are not derived from the curriculum expectations for a course and which constitute alternative programs and/or courses.

If the student requires either accommodations or modified expectations, or both, the relevant information, as described in the following paragraphs, must be recorded in his or her Individual Education Plan (IEP). More detailed information about planning programs for students with special education needs, including students who require alternative programs and/or courses, can be found in *The Individual Education Plan (IEP): A Resource Guide, 2004.* For a detailed discussion of the ministry's requirements for IEPs, see *Individual Education Plans: Standards for Development, Program Planning, and Implementation, 2000.*

Students Requiring Accommodations Only

Some students are able, with certain accommodations, to participate in the regular course curriculum and to demonstrate learning independently. Accommodations allow access to the course without any changes to the knowledge and skills the student is expected to demonstrate. The accommodations required to facilitate the student's learning must be identified in his or her IEP (see IEP Standards, 2000, page 11). A student's IEP is likely to reflect the same accommodations for many, or all, subjects or courses. Providing accommodations to students with special education needs should be the first option considered in program planning. Instruction based on principles of universal design and differentiated instruction focuses on the provision of accommodations to meet the diverse needs of learners. There are three types of accommodations:

- *Instructional accommodations* are changes in teaching strategies, including styles of presentation, methods of organization, or use of technology and multimedia.
- *Environmental accommodations* are changes that the student may require in the classroom and/or school environment, such as preferential seating or special lighting.
- *Assessment accommodations* are changes in assessment procedures that enable the student to demonstrate his or her learning, such as allowing additional time to complete tests or assignments or permitting oral responses to test questions (see page 29 of the IEP Resource Guide, 2004, for more examples).

If a student requires "accommodations only" in mathematics courses, assessment and evaluation of his or her achievement will be based on the appropriate course curriculum expectations and the achievement levels outlined in this document. The IEP box on the student's Provincial Report Card will not be checked, and no information on the provision of accommodations will be included.

Students Requiring Modified Expectations

Some students will require modified expectations, which differ from the regular course expectations. For most students, modified expectations will be based on the regular course curriculum, with changes in the number and/or complexity of the expectations. Modified expectations represent specific, realistic, observable, and measurable achievements and describe specific knowledge and/or skills that the student can demonstrate independently, given the appropriate assessment accommodations.

PROGRAM CONSIDERATIONS FOR ENGLISH LANGUAGE LEARNERS

Young people whose first language is not English enter Ontario secondary schools with diverse linguistic and cultural backgrounds. Some English language learners may have experience of highly sophisticated educational systems, while others may have come from regions where access to formal schooling was limited. All of these students bring a rich array of background knowledge and experience to the classroom, and all teachers must share in the responsibility for their English-language development. Teachers of mathematics must incorporate appropriate adaptations and strategies for instruction and assessment to facilitate the success of the English language learners in their classrooms. These adaptations and strategies include:

- modification of some or all of the course expectations so that they are challenging but attainable for the learner at his or her present level of English proficiency, given the necessary support from the teacher;
- use of a variety of instructional strategies (e.g., extensive use of visual cues, scaffolding, manipulatives, pictures, diagrams, graphic organizers; attention to clarity of instructions);
- modelling of preferred ways of working in mathematics; previewing of textbooks; pre-teaching of key vocabulary; peer tutoring; strategic use of students' first languages);
- use of a variety of learning resources (e.g., visual material, simplified text, bilingual dictionaries, materials that reflect cultural diversity);
- use of assessment accommodations (e.g., granting of extra time; simplification of language used in problems and instructions; use of oral interviews, learning logs, portfolios, demonstrations, visual representations, and tasks requiring completion of graphic organizers or cloze sentences instead of tasks that depend heavily on proficiency in English).

Although the degree of program adaptation required will decrease over time, students who are no longer receiving ESL or ELD support may still need some program adaptations to be successful.

ANTIDISCRIMINATION EDUCATION IN MATHEMATICS

To ensure that all students in the province have an equal opportunity to achieve their full potential, the curriculum must be free from bias, and all students must be provided with a safe and secure environment, characterized by respect for others, that allows them to participate fully and responsibly in the educational experience.

Learning activities and resources used to implement the curriculum should be inclusive in nature, reflecting the range of experiences of students with varying backgrounds, abilities, interests, and learning styles. They should enable students to become more sensitive to the diverse cultures and perceptions of others, including Aboriginal peoples. By discussing aspects of the history of mathematics, teachers can help make students aware of the various cultural groups that have contributed to the evolution of mathematics over the centuries. Finally, students need to recognize that ordinary people use mathematics in a variety of everyday contexts, both at work and in their daily lives.

Connecting mathematical ideas to real-world situations through learning activities can enhance students' appreciation of the role of mathematics in human affairs, in areas including health, science, and the environment. Students can be made aware of the use of mathematics in contexts such as sampling and surveying and the use of statistics to analyse trends. Recognizing the importance of mathematics in such areas helps motivate students to learn and also provides a foundation for informed, responsible citizenship.

Teachers should have high expectations for all students. To achieve their mathematical potential, however, different students may need different kinds of support. Some boys, for example, may need additional support in developing their literacy skills in order to complete mathematical tasks effectively. For some girls, additional encouragement to envision themselves in careers involving mathematics may be beneficial. For example, teachers might consider providing strong role models in the form of female guest speakers who are mathematicians or who use mathematics in their careers.

LITERACY AND INQUIRY/RESEARCH SKILLS

Literacy skills can play an important role in student success in mathematics courses. Many of the activities and tasks students undertake in mathematics courses involve the use of written, oral, and visual communication skills. For example, students use language to record their observations, to explain their reasoning when solving problems, to describe their inquiries in both informal and formal contexts, and to justify their results in small- group conversations, oral presentations, and written reports. The language of mathematics includes special terminology. The study of mathematics consequently encourages students to use language with greater care and precision and enhances their ability to communicate effectively.

The Ministry of Education has facilitated the development of materials to support literacy instruction across the curriculum. Helpful advice for integrating literacy instruction in mathematics courses may be found in the following resource documents:

- *Think Literacy: Cross-Curricular Approaches, Grades 7–12, 2003*
- *Think Literacy: Cross-Curricular Approaches, Grades 7–12 – Mathematics: Subject-Specific Examples, Grades 10–12, 2005*

In all courses in mathematics, students will develop their ability to ask questions and to plan investigations to answer those questions and to solve related problems. Students need to learn a variety of research methods and inquiry approaches in order to carry out these investigations and to solve problems, and they need to be able to select the methods that are most appropriate for a particular inquiry. Students learn how to locate relevant information from a variety of sources, such as statistical databases, newspapers, and reports. As they advance through the grades, students will be expected to use such sources with increasing sophistication. They will also be expected to distinguish between primary and secondary sources, to determine their validity and relevance, and to use them in appropriate ways.

THE ROLE OF INFORMATION AND COMMUNICATION TECHNOLOGY IN MATHEMATICS

Information and communication technologies (ICT) provide a range of tools that can significantly extend and enrich teachers' instructional strategies and support students' learning in mathematics. Teachers can use ICT tools and resources both for whole-class instruction and to design programs that meet diverse student needs. Technology can help to reduce the time spent on routine mathematical tasks, allowing students to devote more of their efforts to thinking and concept development. Useful ICT tools include simulations, multimedia resources, databases, sites that give access to large amounts of statistical data, and computer-assisted learning modules.

Applications such as databases, spreadsheets, dynamic geometry software, dynamic statistical software, graphing software, computer algebra systems (CAS), word-processing software, and presentation software can be used to support various methods of inquiry in mathematics. Technology also makes possible simulations of complex systems that can be useful for problem-solving purposes or when field studies on a particular topic are not feasible.

Information and communications technologies can be used in the classroom to connect students to other schools, at home and abroad, and to bring the global community into the local classroom.

Although the Internet is a powerful electronic learning tool, there are potential risks attached to its use. All students must be made aware of issues of Internet privacy, safety, and responsible use, as well as of the ways in which this technology is being abused – for example, when it is used to promote hatred.

Teachers, too, will find the various ICT tools useful in their teaching practice, both for whole class instruction and for the design of curriculum units that contain varied approaches to learning to meet diverse student needs.

CAREER EDUCATION IN MATHEMATICS

Teachers can promote students' awareness of careers involving mathematics by exploring applications of concepts and providing opportunities for career-related project work. Such activities allow students the opportunity to investigate mathematics-related careers compatible with their interests, aspirations, and abilities.

Students should be made aware that mathematical literacy and problem solving are valuable assets in an ever-widening range of jobs and careers in today's society. The knowledge and skills students acquire in mathematics courses are useful in fields such as science, business, engineering, and computer studies; in the hospitality, recreation, and tourism industries; and in the technical trades.

THE ONTARIO SKILLS PASSPORT AND ESSENTIAL SKILLS

Teachers planning programs in mathematics need to be aware of the purpose and benefits of the Ontario Skills Passport (OSP). The OSP is a bilingual web-based resource that enhances the relevancy of classroom learning for students and strengthens school-work connections. The OSP provides clear descriptions of Essential Skills such as Reading Text, Writing, Computer Use, Measurement and Calculation, and Problem Solving and includes an extensive database of occupation-specific workplace tasks that illustrate how workers use these skills on the job. The Essential Skills are transferable, in that they are used in virtually all occupations. The OSP also includes descriptions of important work habits, such as working safely, being reliable, and providing excellent customer service. The OSP is designed to help employers assess and record students' demonstration of these skills and work habits during their cooperative education placements. Students can use the OSP to identify the skills and work habits they already have, plan further skill development, and show employers what they can do.

The skills described in the OSP are the Essential Skills that the Government of Canada and other national and international agencies have identified and validated, through extensive research, as the skills needed for work, learning, and life. These Essential Skills provide the foundation for learning all other skills and enable people to evolve with their jobs and adapt to workplace change. For further information on the OSP and the Essential Skills, visit: http://skills.edu.gov.on.ca.

COOPERATIVE EDUCATION AND OTHER FORMS OF EXPERIENTIAL LEARNING

Cooperative education and other workplace experiences, such as job shadowing, field trips, and work experience, enable students to apply the skills they have developed in the classroom to real-life activities. Cooperative education and other workplace experiences also help to broaden students' knowledge of employment opportunities in a wide range of fields, including science and technology, research in the social sciences and humanities, and many forms of business administration. In addition, students develop their understanding of workplace practices, certifications, and the nature of employer-employee relationships.

Cooperative education teachers can support students taking mathematics courses by maintaining links with community-based businesses and organizations, and with colleges and universities, to ensure students studying mathematics have access to hands-on experiences that will reinforce the knowledge and skills they have gained in school. Teachers of mathematics can support their students' learning by providing opportunities for experiential learning that will reinforce the knowledge and skills they have gained in school.

Health and safety issues must be addressed when learning involves cooperative education and other workplace experiences. Teachers who provide support for students in workplace learning placements need to assess placements for safety and ensure students understand the importance of issues relating to health and safety in the workplace. Before taking part in workplace learning experiences, students must acquire the knowledge and skills needed for safe participation. Students must understand their rights to privacy and confidentiality as outlined in the Freedom of Information and Protection of Privacy Act. They have the right to function in an environment free from abuse and harassment, and they need to be aware of harassment and abuse issues in establishing boundaries for their own personal safety. They should be informed about school and community resources and school policies and reporting procedures with regard to all forms of abuse and harassment.

Policy/Program Memorandum No. 76A, "Workplace Safety and Insurance Coverage for Students in Work Education Programs" (September 2000), outlines procedures for ensuring the provision of Health and Safety Insurance Board coverage for students who are at least 14 years of age and are on placements of more than one day. (A one-day job-shadowing or job-twinning experience is treated as a field trip.) Teachers should also be aware of the minimum age requirements outlined in the Occupational Health and Safety Act for persons to be in or to be working in specific workplace settings.

All cooperative education and other workplace experiences will be provided in accordance with the ministry's policy document entitled *Cooperative Education and Other Forms of Experiential Learning: Policies and Procedures for Ontario Secondary Schools, 2000*.

PLANNING PROGRAM PATHWAYS AND PROGRAMS LEADING TO A SPECIALIST HIGH-SKILLS MAJOR

Mathematics courses are well suited for inclusion in programs leading to a Specialist High-Skills Major (SHSM) or in programs designed to provide pathways to particular apprenticeship or workplace destinations. In an SHSM program, mathematics courses can be bundled with other courses to provide the academic knowledge and skills important to particular industry sectors and required for success in the workplace and postsecondary education, including apprenticeship. Mathematics courses may also be combined with cooperative education credits to provide the workplace experience required for SHSM programs and for various program pathways to apprenticeship and workplace destinations. (SHSM programs would also include sector-specific learning opportunities offered by employers, skills-training centres, colleges, and community organizations.)

HEALTH AND SAFETY IN MATHEMATICS

Although health and safety issues are not normally associated with mathematics, they may be important when learning involves fieldwork or investigations based on experimentation. Out-of-school fieldwork can provide an exciting and authentic dimension to students' learning experiences. It also takes the teacher and students out of the predictable classroom environment and into unfamiliar settings. Teachers must preview and plan activities and expeditions carefully to protect students' health and safety.

APPENDIX 1 – ACHIEVEMENT CHART

Achievement Chart – Mathematics, Grades 9–12

Categories	50–59% (Level 1)	60–69% (Level 2)	70–79% (Level 3)	80–100% (Level 4)
Knowledge and Understanding *Subject-specific content acquired in each course (knowledge), and the comprehension of its meaning and significance (understanding)*				
	The student:			
Knowledge of content (e.g., facts, terms, procedural skills, use of tools)	– demonstrates limited knowledge of content	– demonstrates some knowledge of content	– demonstrates considerable knowledge of content	– demonstrates thorough knowledge of content
Understanding of mathematical concepts	– demonstrates limited understanding of concepts	– demonstrates some understanding of concepts	– demonstrates considerable understanding of concepts	– demonstrates thorough understanding of concepts
Thinking *The use of critical and creative thinking skills and/or processes**				
	The student:			
Use of planning skills – understanding the problem (e.g., formulating and interpreting the problem, making conjectures) – making a plan for solving the problem	– uses planning skills with limited effectiveness	– uses planning skills with some effectiveness	– uses planning skills with considerable effectiveness	– uses planning skills with a high degree of effectiveness
Use of processing skills – carrying out a plan (e.g., collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions) – looking back at the solution (e.g., evaluating reasonableness, making convincing arguments, reasoning, justifying, proving, reflecting)	– uses processing skills with limited effectiveness	– uses processing skills with some effectiveness	– uses processing skills with considerable effectiveness	– uses processing skills with a high degree of effectiveness
Use of critical/creative thinking processes (e.g., problem solving, inquiry)	– uses critical/creative thinking processes with limited effectiveness	– uses critical/creative thinking processes with some effectiveness	– uses critical/creative thinking processes with considerable effectiveness	– uses critical/creative thinking processes with a high degree of effectiveness

* The processing skills and critical/creative thinking processes in the Thinking category include some but not all aspects of the *mathematical processes* described on pages 12–16 of this document. Some aspects of the mathematical processes relate to the other categories of the achievement chart.

Categories	50–59% (Level 1)	60–69% (Level 2)	70–79% (Level 3)	80–100% (Level 4)
Communication *The conveying of meaning through various forms*				
	The student:			
Expression and organization of ideas and mathematical thinking (e.g., clarity of expression, logical organization), using oral, visual, and written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; concrete materials)	– expresses and organizes mathematical thinking with limited effectiveness	– expresses and organizes mathematical thinking with some effectiveness	– expresses and organizes mathematical thinking with considerable effectiveness	– expresses and organizes mathematical thinking with a high degree of effectiveness
Communication for different audiences (e.g., peers, teachers) and purposes (e.g., to present data, justify a solution, express a mathematical argument) in oral, visual, and written forms	– communicates for different audiences and purposes with limited effectiveness	– communicates for different audiences and purposes with some effectiveness	– communicates for different audiences and purposes with considerable effectiveness	– communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions, vocabulary, and terminology of the discipline (e.g., terms, symbols) in oral, visual, and written forms	– uses conventions, vocabulary, and terminology of the discipline with limited effectiveness	– uses conventions, vocabulary, and terminology of the discipline with some effectiveness	– uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness	– uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness
Application *The use of knowledge and skills to make connections within and between various contexts*				
	The student:			
Application of knowledge and skills in familiar contexts	– applies knowledge and skills in familiar contexts with limited effectiveness	– applies knowledge and skills in familiar contexts with some effectiveness	– applies knowledge and skills in familiar contexts with considerable effectiveness	– applies knowledge and skills in familiar contexts with a high degree of effectiveness
Transfer of knowledge and skills to new contexts	– transfers knowledge and skills to new contexts with limited effectiveness	– transfers knowledge and skills to new contexts with some effectiveness	– transfers knowledge and skills to new contexts with considerable effectiveness	– transfers knowledge and skills to new contexts with a high degree of effectiveness
Making connections within and between various contexts (e.g., connections between concepts, representations, and forms within mathematics; connections involving use of prior knowledge and experience; connections between mathematics, other disciplines, and the real world)	– makes connections within and between various contexts with limited effectiveness	– makes connections within and between various contexts with some effectiveness	– makes connections within and between various contexts with considerable effectiveness	– makes connections within and between various contexts with a high degree of effectiveness

APPENDIX 2 - LEARNING SKILLS & WORK HABITS

LEARNING SKILLS AND WORK HABITS IN GRADES 1 TO 12

Learning Skills and Work Habits	Sample Behaviours
Responsibility	The student: • fulfils responsibilities and commitments within the learning environment; • completes and submits class work, homework, and assignments according to agreed-upon timelines; • takes responsibility for and manages own behaviour.
Organization	The student: • devises and follows a plan and process for completing work and tasks; • establishes priorities and manages time to complete tasks and achieve goals; • identifies, gathers, evaluates, and uses information, technology, and resources to complete tasks.
Independent Work	The student: • independently monitors, assesses, and revises plans to complete tasks and meet goals; • uses class time appropriately to complete tasks; • follows instructions with minimal supervision.
Collaboration	The student: • accepts various roles and an equitable share of work in a group; • responds positively to the ideas, opinions, values, and traditions of others; • builds healthy peer-to-peer relationships through personal and media-assisted interactions; • works with others to resolve conflicts and build consensus to achieve group goals; • shares information, resources, and expertise and promotes critical thinking to solve problems and make decisions.
Initiative	The student: • looks for and acts on new ideas and opportunities for learning; • demonstrates the capacity for innovation and a willingness to take risks; • demonstrates curiosity and interest in learning; • approaches new tasks with a positive attitude; • recognizes and advocates appropriately for the rights of self and others.
Self-regulation	The student: • sets own individual goals and monitors progress towards achieving them; • seeks clarification or assistance when needed; • assesses and reflects critically on own strengths, needs, and interests; • identifies learning opportunities, choices, and strategies to meet personal needs and achieve goals; • perseveres and makes an effort when responding to challenges.

APPENDIX 3 - RESOURCE LIST

The Ontario Curriculum Grade 11 and 12 Mathematics (revised), published 2007

Ontario Ministry of Education. *Think Literacy: Cross Curricular Approaches, Grades 7-12; Mathematics 10-12*.